LAWRENCE
LIVERMORE
NATIONAL
LABORATORY

UCRL-CONF-200773

Elastic and Poroelastic Analysis of Thomsen Parameters for Seismic Waves in Finely Layered VTI Media

J. G. Berryman

November 5, 2003

Society of Exploration Geophysicists
Dallas, TX, United States
October 27, 2003 through October 31, 2003

UCRL-CONF-200773 **October 30, 2003**

Elastic and Poroelastic Analysis of Thomsen Parameters for Seismic Waves in Finely Layered VTI Media

James G. Berryman

University of California, LLNL

This work was performed under the auspices of the U.S. Department of Energy by the University of California, Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

Dallas, TX, October 26–31, 2003

Elastic and Poroelastic Analysis of Thomsen Parameters for Seismic Waves in Finely Layered VTI Media

James G. Berryman
University of California, LLNL
and Stanford University

OUTLINE

- Statement of the Problem: Shear Dependence on Pore Fluids
- Elasticity in VTI Media
- Backus Averaging for Finely Layered Media
- Thomsen Parameters
- Uniaxial Shear Strain: Its Special Role for Pore Fluids
- Analysis of Wave Dispersion
- Some Examples

Notation for Elastic Stiffness in VTI Media

With σ_{ij} being the ij component of stress, and

e_{kl} being the kl component of strain:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} a & b & f & & & \\ b & a & f & & & \\ f & f & c & & & \\ & & & 2l & & \\ & & & & 2l & \\ & & & & & 2m \end{pmatrix} \begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \\ e_{23} \\ e_{31} \\ e_{12} \end{pmatrix},$$

where $a = b + 2m$ (e.g., Musgrave, 1970; Auld, 1973).

Indices i, j, k, l range from 1 to 3 in Cartesian coordinates.

Following Backus (1962), we suppose that the region of interest is composed of fine layers having isotropic elastic constants, $\lambda(z)$ and $\mu(z)$, being functions of depth z. Then, the average over an arbitrary stack of such layers can be computed using a layer averaging method. This involves a Legendre transform that I will not present here. We use the layer averaging operator symbolized, for example, by brackets

$$\langle\mu\rangle \equiv \frac{1}{D}\int_0^D \mu(z)dz.$$

where D is the depth of the stack of layers.

The elastic anisotropy coefficients are then related to the layer parameters by the following expressions:

$$c = \left\langle \frac{1}{\lambda + 2\mu} \right\rangle^{-1}, \qquad \text{and} \qquad l = \left\langle \frac{1}{\mu} \right\rangle^{-1},$$

$$f = c \left\langle \frac{\lambda}{\lambda + 2\mu} \right\rangle,$$

$$a = \frac{f^2}{c} + 4m - 4 \left\langle \frac{\mu^2}{\lambda + 2\mu} \right\rangle,$$

$$m = \langle \mu \rangle, \qquad \text{and} \qquad b = a - 2m.$$

How Do Fluids Affect These Constants?

Within each isotropic layer, Gassmann says that the shear modulus μ is independent of all fluids present.

So all the dependence on fluids in this layered model comes in through the other Lamé constant

$$\lambda = K - 2\mu/3,$$

where K is the bulk modulus. Depending on the situation, K can be the drained bulk modulus K_d, or it can be the undrained bulk modulus K_u.

Gassmann's Equation for Bulk Modulus

Gassmann's well-known result for fluid-substitution is:

$$K_u = K_d/(1 - \alpha B),$$

where K_d is the drained bulk modulus, $\alpha = 1 - K_d/K_s$ is the Biot-Willis or effective stress coefficient with K_s being a measure of the grain bulk moduli, while B is Skempton's coefficient, containing all the relevant information about the fluid moduli and porosity.

Note that $1/(1 - \alpha B)$ is a magnification factor.

Thomsen's Parameters for Weak Anisotropy

The Thomsen (1986) parameters ϵ, δ, and γ are related to these stiffness coefficients by

$$\epsilon \equiv \frac{a-c}{2c},$$

$$\delta \equiv \frac{(f+l)^2-(c-l)^2}{2c(c-l)},$$

$$\gamma \equiv \frac{m-l}{2l}.$$

Elastic Stiffness Matrix in VTI Media

With σ_{ij} being the ij component of stress, and e_{kl} being the kl component of strain:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} a & b & f & & & \\ b & a & f & & & \\ f & f & c & & & \\ & & & 2l & & \\ & & & & 2l & \\ & & & & & 2m \end{pmatrix} \begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \\ e_{23} \\ e_{31} \\ e_{12} \end{pmatrix},$$

where $a = b + 2m$ (e.g., Musgrave, 1970; Auld, 1973).

Indices i, j, k, l range from 1 to 3 in Cartesian coordinates.

We can immediately write down four singular vectors (or eigenvectors):

$$\begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix}, \quad \begin{pmatrix}0\\0\\0\\0\\1\\0\end{pmatrix}, \quad \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}, \quad \begin{pmatrix}1\\-1\\0\\0\\0\\0\end{pmatrix},$$

and their corresponding singular values (eigenvalues), are respectively: $2l$, $2l$, $2m$, and $a - b = 2m$.

All four correspond to shear modes of the system.

Uniaxial shear strain can be applied to this system in the form:

$$\begin{pmatrix} 1 \\ 1 \\ -2 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Then, the associated energy per unit volume for such an excitation can be easily determined.

Uniaxial Shear Strain: Energy per Unit Volume

Ignoring the parts of the matrix not relevant, we now have:

$$E_v = v^T \begin{pmatrix} a & b & f \\ b & a & f \\ f & f & c \end{pmatrix} v,$$

where the normalized vector of interest is

$$v^T = \begin{pmatrix} 1 & 1 & -2 \end{pmatrix} / \sqrt{6},$$

and

$$E_v = 2[a - m + c - 2f]/3 \equiv 2G_{eff}.$$

Uniaxial Strain: An Effective Shear Modulus

Thus, based on energy alone, we can justify defining

$$G_{eff} \equiv [a - m + c - 2f]/3.$$

For this and several other reasons I will not have time to discuss, G_{eff} acts like an effective shear modulus and it is the only one of the five shear moduli that ever contains information about pore fluids.

The general behavior of seismic waves in anisotropic media is well known, and the equations are derived in many places including Berryman (1979) and Thomsen (1986). The results are

$$\rho\omega_{\pm}^2 = \frac{1}{2}\Big[(a+l)k_1^2 + (c+l)k_3^2$$

$$\pm\sqrt{[(a-l)k_1^2 - (c-l)k_3^2]^2 + 4(f+l)^2k_1^2k_3^2}\Big],$$

for compressional (+) and vertically polarized shear (−) waves and

Dispersion Relations (continued)

$$\rho\omega_s^2 = mk_1^2 + lk_3^2,$$

for horizontally polarized shear waves, where ρ is the overall density, ω is the angular frequency, k_1 and k_3 are the horizontal and vertical wavenumbers (respectively), and the velocities are given simply by $v = \omega/k$ with $k = \sqrt{k_1^2 + k_3^2}$.

The SH wave depends only on elastic parameters l and m, which are not dependent in any way on layer λ and, therefore, play no role in the poroelastic analysis. Thus, we can safely ignore SH except when we want to check for shear wave splitting (bi-refringence) – in which case the SH results will be useful for the comparisons.

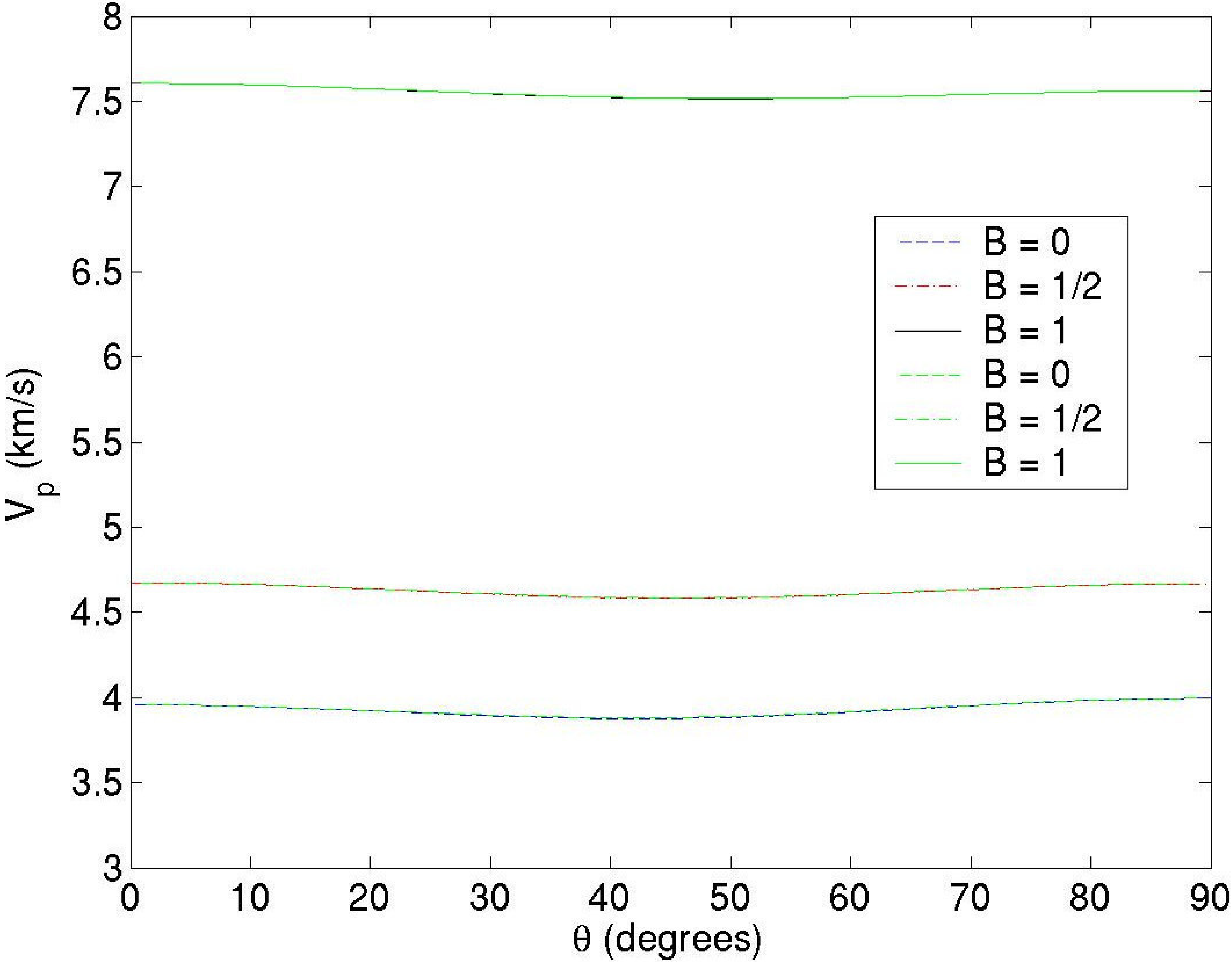
$$\rho\omega_+^2 \equiv ak_1^2 + ck_3^2 - \Delta,$$

and

$$\rho\omega_-^2 \equiv lk^2 + \Delta,$$

with Δ determined approximately by

$$\Delta \simeq \frac{[(a-l)(c-l) - (f+l)^2]}{(a-l)/k_3^2 + (c-l)/k_1^2}.$$


8
7.5
7
6.5
6
5.5
5
4.5
4
3.5
3
0
10
20
30
40
50
60
70
80
90
V_p (km/s)
θ (degrees)
B = 0
B = 1/2
B = 1
B = 0
B = 1/2
B = 1

Recall that

$$(a - l)(c - l) - (f + l)^2 = 2c(c - l)(\epsilon - \delta).$$

We can also rewrite the first elasticity factor in the denominator as $a - l = (c - l)[1 + 2c\epsilon/(c - l)]$. Combining these results in the limit of $k_1^2 \to 0$ (for relatively small horizontal offset), we find that

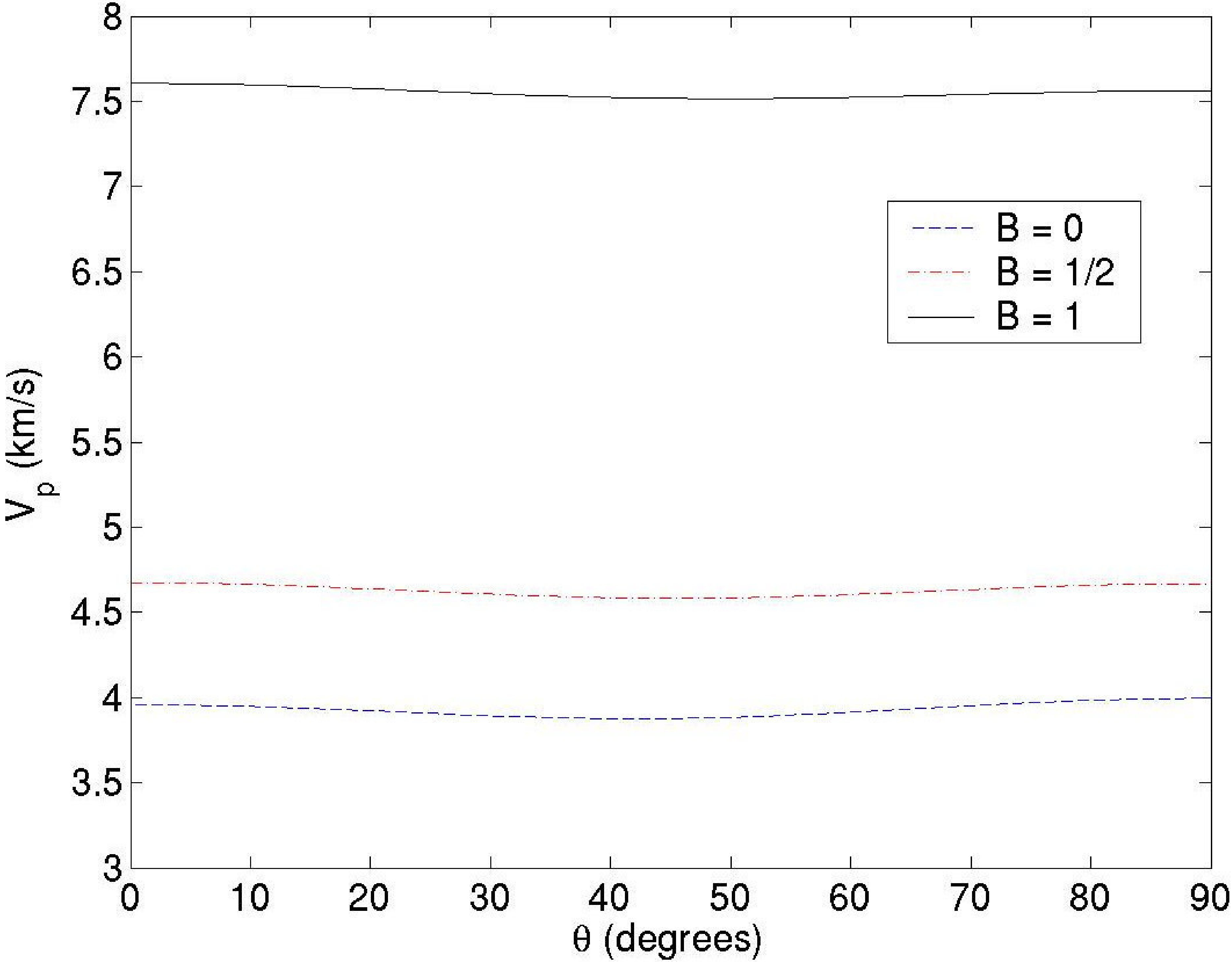
$$\rho\omega_+^2 \simeq ck^2 + 2c\delta k_1^2,$$

and

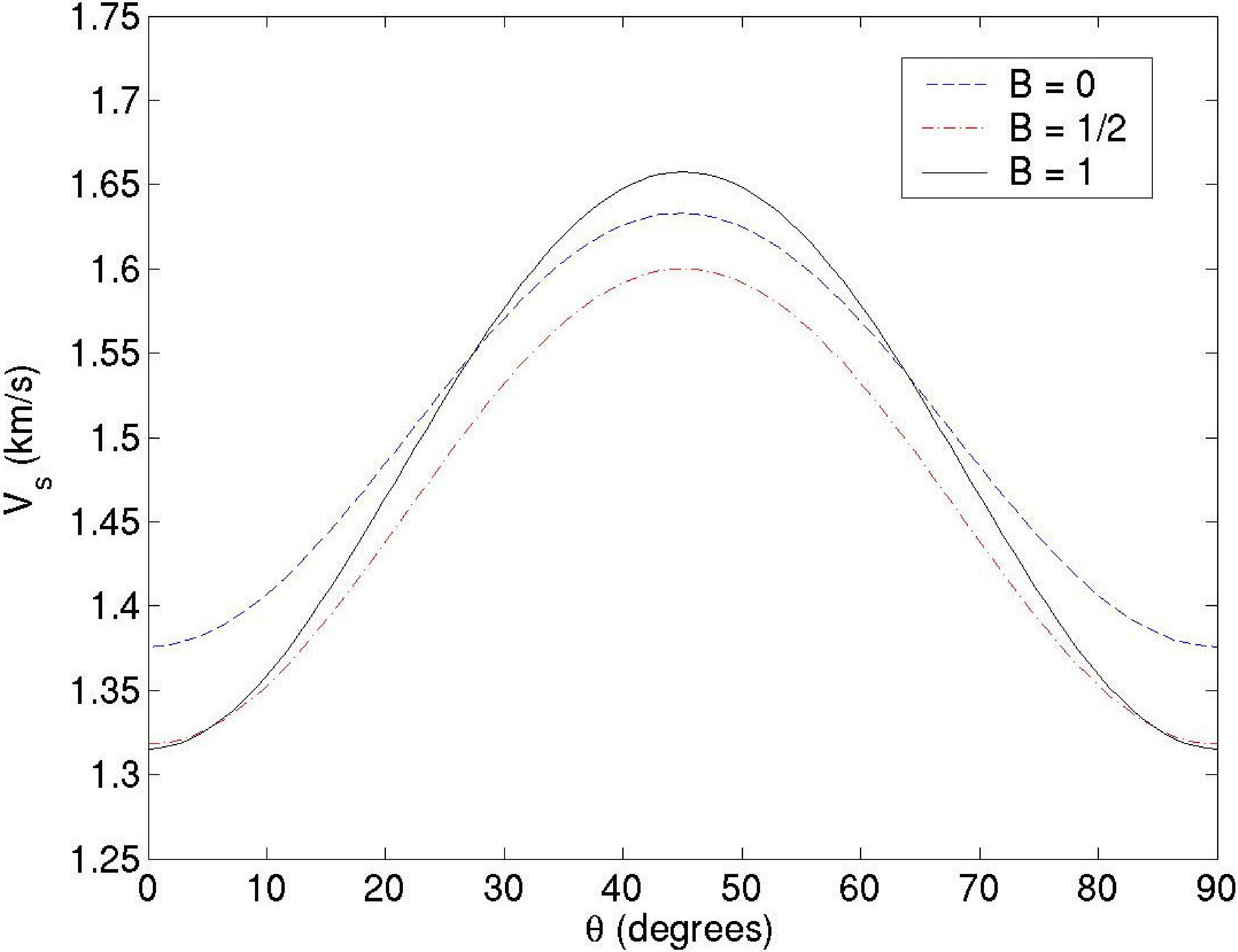
$$\rho\omega_-^2 \simeq lk^2 + 2c(\epsilon - \delta)k_1^2,$$

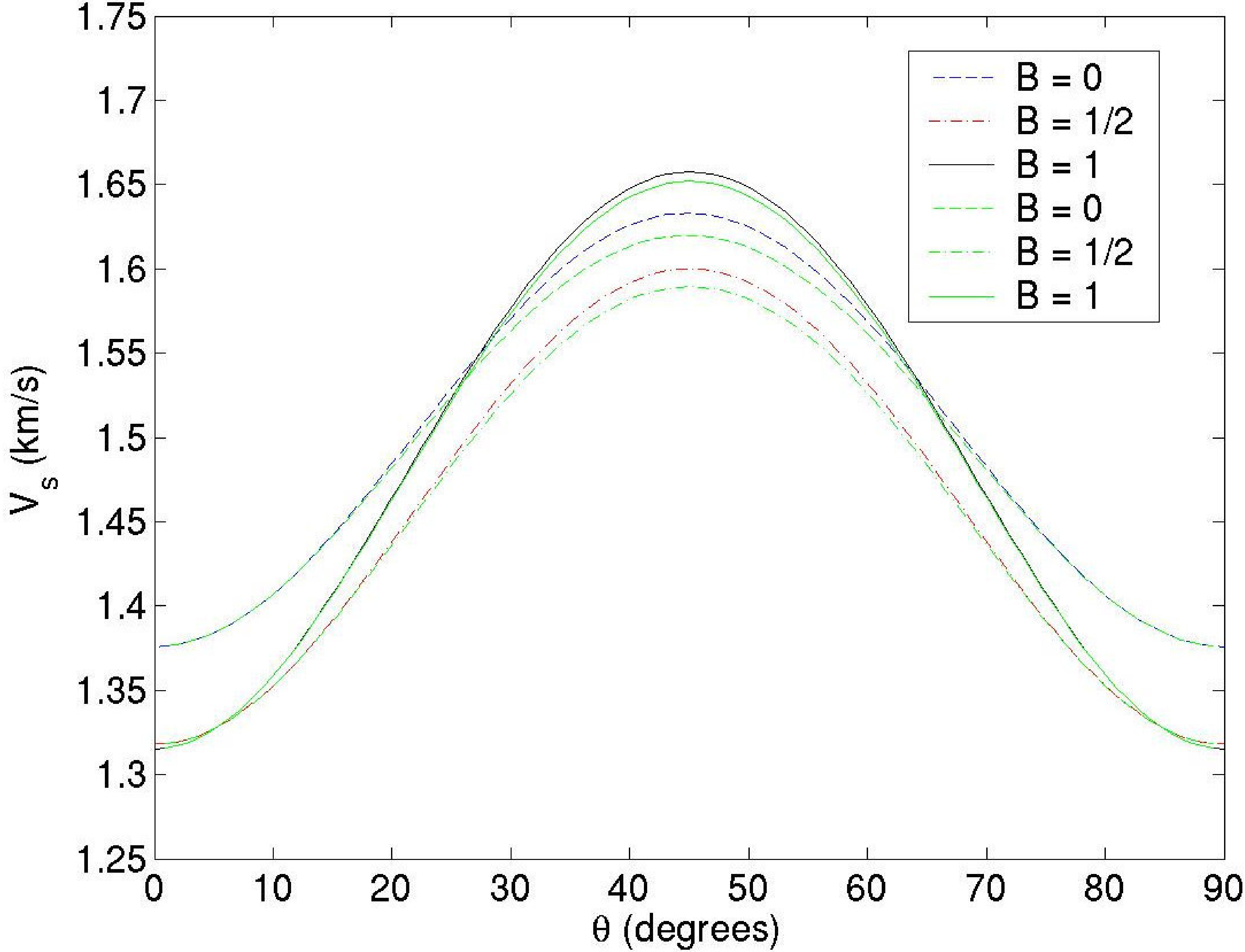
since, in this limit, we have $\Delta \simeq 2c(\epsilon - \delta)k_1^2$.

Improved approximations to any desired order can be obtained with only a little more effort by keeping more terms in the expansion.



8
7.5
7
6.5
6
5.5
5
4.5
4
3.5
3
0
10
20
30
40
50
60
70
80
90
V_p (km/s)
θ (degrees)
B = 0
B = 1/2
B = 1


B = 0
B = 1/2
B = 1
V_s (km/s)
θ (degrees)
1.75
1.7
1.65
1.6
1.55
1.5
1.45
1.4
1.35
1.3
1.25
0
10
20
30
40
50
60
70
80
90


B = 0
B = 1/2
B = 1
B = 0
B = 1/2
B = 1
V_s (km/s)
θ (degrees)
1.75
1.7
1.65
1.6
1.55
1.5
1.45
1.4
1.35
1.3
1.25
0
10
20
30
40
50
60
70
80
90

CONCLUSIONS

- Of the five shear moduli of a VTI system, only G_{eff} as defined here can ever contain information about pore fluids.
- Pore fluids have their biggest effects at $\theta = 45°$, on both quasi-P and quasi-SV waves in layered VTI media.
- The stiffening effects of pore fluids can be substantial if all the necessary conditions are right.
- These results show that the observed moveout of wave velocities with angle can in some circumstances be a strong indicator of trapped liquids.